

## Emulating quantum cubic nonlinearity

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Unitary non-Gaussian nonlinearity is one of the key components required for quantum computation and other developing applications of quantum information processing. Sufficient operation of this kind is still not available, but it can be approximatively implemented with the help of a specifically engineered resource state constructed from individual photons. We present experimental realization and thorough analysis of such quantum resource states and confirm that the state does indeed possess properties of a state produced by unitary dynamics driven by cubic nonlinearity.

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### I. INTRODUCTION

Nonlinear interactions capable of manipulating the quantum state of the harmonic oscillators form a very challenging area of recent development in the field of modern quantum physics. Handling these interactions is necessary not only for the understanding of quantum nonlinear dynamics of the harmonic oscillators, but also for achieving the standing long-term goal of quantum information—the universal quantum computation [1,2]. The operations needed are unitary, and both Gaussian and non-Gaussian [3]. For a harmonic oscillator representing a single mode of electromagnetic radiation, Gaussian operations are relatively easy to obtain, but unitary single-mode non-Gaussian nonlinearities are either not available or are too weak to have an observable quantum effect. For other physical systems, such as cold atoms [4] or trapped ions [5], the non-Gaussian operations could be implemented using additional anharmonic potentials, but multimode Gaussian operations are hard to come by.

Quantum nonlinear operations for light can be obtained by letting the system interact with individual atoms, ions [6,7], or similar solid-state physical systems [8] and measuring the discrete system afterwards. In this way, highly nonclassical superposed coherent states were recently realized [9]. These quantum states possess strong nonlinear properties, and they were not previously observed in the trapped ions [7], the circuit cavity electrodynamic [10], and in the optical continuous variables (CV) experiments with traveling light [11,12]. However, of these systems, only the last one currently allows implementation of deterministic Gaussian operations and measurements [13–16], which are needed for deterministic measurement-induced implementation of high-order nonlinearities [17,18]. Considering that recently single-photon detectors [3] were used to prepare states with nonlinear properties at least approaching those of atomic and solid-state systems [19], the toolbox of CV quantum optics has everything it needs for tests of unitary nonlinear dynamics. Please note, there is a difference between the CV quantum optics and its discrete counterpart, relying on encoding qubits into individual photons [20,21]. In both, the desired nonlinearity can be obtained from measurements, and highly nonlinear gates have indeed been implemented for single photons [22]. However, the current level of technology does not allow discrete quantum optics experiments to be truly

deterministic, as all measurements need to be performed in coincidence basis.

In principle, to realize an arbitrary unitary operation of a quantum harmonic oscillator, it is sufficient to have access to the quantum cubic nonlinearity [1,23]. Cubic nonlinearity is represented by a Hamiltonian  $\hat{H} \propto \hat{x}^3$  [17], where  $\hat{x} = (\hat{a} + \hat{a}^\dagger)/\sqrt{2}$  is the position operator of the quantum harmonic oscillator [ $\hat{a}$  is the annihilation operator, and the momentum operator is similarly defined as  $\hat{p} = (\hat{a} - \hat{a}^\dagger)/(i\sqrt{2})$ ]. As of now, neither quantum cubic nonlinearity, nor quantum states produced by it (cubic states), have been observed on any experimental platform. Beginning from a ground state, even the weak cubic interaction generates highly nonclassical states [24]. However, the nonclassicality of these states lies in the superposition of  $|1\rangle$  and  $|3\rangle$  ( $|1&3\rangle$  for shorthand), and it is unfortunately masked by the superposition of  $|1&3\rangle$  with the dominant ground state  $|0\rangle$  [18], especially considering its fragility with regard to damping of the oscillator. It is therefore challenging not only to generate and detect these states, but also to understand and verify their nonclassical features.

A nonlinear gate can be deterministically implemented by coupling a specifically prepared ancillary state to the unknown target via the Gaussian quantum nondemolition coupling. The ancilla is then measured and the obtained value  $q$  is used to drive nonlinear feed forward in the form of pair of displacements proportional to  $q$  and  $q^2$  performed on the target state. See Ref. [18] for more details. This approach was initially discussed in [17,24,25] with the ideal state, which is currently experimentally unfeasible. To remedy this issue, an *approximative* weak cubic state, described as a superposition of Fock states  $|0\rangle$ ,  $|1\rangle$ , and  $|3\rangle$ , was recently proposed [18]. In this paper we present the experimental, completely heralded preparation of this state together with analysis of its nontrivial nonclassical properties.

### II. CUBIC STATE

The ideal cubic state, which can be used as a resource for the nonlinear cubic gate, can be expressed as  $\int e^{-i\chi_0 \hat{x}^3} |x\rangle dx$ . Note that normalization factors are omitted in this paper unless otherwise noted. The cubic state can be obtained by applying cubic nonlinear interaction  $\hat{U}(\chi_0) = \exp(-i\chi_0 \hat{x}^3)$  to an infinitely squeezed state. Due to general inaccessibility of

a cubic nonlinear operation, any physical realization of the state needs to be some kind of approximation. For weak cubic nonlinearity and finite energy, the state can be approximated by  $\hat{S}(-r)(1 - i\chi\hat{x}^3)|0\rangle$  [18]. Here, the cubic nonlinearity  $\chi$  is given by  $\chi = \chi_0 e^{3r}$ , and  $\hat{S}(-r) = \exp[-(ir/2)(\hat{x}\hat{p} + \hat{p}\hat{x})]$  is a squeezing operation, a Gaussian operation, which can be considered feasible and highly accessible in contemporary experimental practice [13–16]. The squeezing operation does not affect the cubic behavior of the state and therefore can be omitted in our first attempts to implement the cubic operation. The approximative weak cubic state can be then expressed in the Fock space as

$$|\psi_{id}\rangle = (1 - i\chi\hat{x}^3)|0\rangle = |0\rangle - i\frac{\chi\sqrt{15}}{2\sqrt{2}}|1&3\rangle, \quad (1)$$

where  $|1&3\rangle = (\sqrt{3}|1\rangle + \sqrt{2}|3\rangle)/\sqrt{5}$ . It is a specific superposition of zero, one, and three photons, but it can also be viewed as a superposition of vacuum  $|0\rangle$  with a state  $|1&3\rangle$ , which in itself is an approximation of odd superposition of coherent states. The vacuum contribution results from the first term of the unitary evolution  $\hat{U}(\chi) \approx 1 - i\chi\hat{x}^3$ . It is an important term for the function of the deterministic cubic phase gate, but at the same time it masks the nonclassical features of the state  $|1&3\rangle$ .

The cubic state (1) is generated by means of the setup depicted in Fig. 1. The nondegenerate optical parametric oscillator (NOPO) generates an entangled two-mode squeezed state  $\sum_{n=0}^{\infty} \lambda^n |n\rangle_i |n\rangle_s$ . The *idler* mode *i* is then split into three by a pair of beamsplitters, after which the states of the three modes are displaced in a phase space by amplitudes  $\alpha = 1.55\lambda e^{i90^\circ}$ ,  $\beta = 1.19\lambda e^{i311^\circ}$ , and  $\gamma = 1.19\lambda e^{i229^\circ}$ . Finally, each of the modes impinges on the avalanche photodiode (APD). Simultaneous detection of a photon by the three detectors then heralds approximative preparation of the *signal*

mode *s* in the state

$$\sum_{n=0}^3 \lambda^n [(1, 1, 1)_{i12} \hat{D}_1(\alpha) \hat{D}_2(\beta) \hat{D}_3(\gamma) \hat{U}_{BS}^{12} \hat{U}_{BS}^{1i} |n, 0, 0\rangle_{i12}] |n\rangle_s, \quad (2)$$

where  $\hat{D}_k(\cdot)$  represents the displacement operation on mode *k*,  $\hat{U}_{BS}^{kl}$  represents the beamsplitter between modes *k* and *l*, and subscripts 1 and 2 describe the ancillary modes. For the suitable choice of  $\lambda$ , this state turns into the required superposition (1). Please note that the state is prepared from the higher Fock number contributions of a single two-mode state, and not from several single photons as in [26]. In this sense it is actually more reminiscent of the proposal relying on repeated combinations of displacements and photon subtractions performed on a single-mode squeezed light [27]. As a consequence, the photons forming the state are indistinguishable. There are also no problems with mode structure, because the heralded state is measured by homodyne detection, the local oscillator of which perfectly defines the measured mode. Any multimode effects, arising, for example, from imperfect coincidence of the APDs, therefore directly translate to reduction of the overall quality of the produced state.

### III. THE EXPERIMENT

The light source is a continuous-wave Ti:sapphire laser of 860 nm. With around 20 mW of pump beam of 430 nm, a two-mode squeezed vacuum is generated from a NOPO, which contains a periodically poled KTiOPO<sub>4</sub> crystal as an optical nonlinear medium. The pump beam is generated by second harmonic generation of the fundamental beam and frequency-shifted with an acousto-optic modulator by around 600 MHz (equal to the free spectral range of NOPO,  $\Delta\omega$ ). As a result, photon pairs of frequency  $\omega$  (*signal*) and  $\omega + \Delta\omega$  (*idler*) are obtained ( $\omega$  corresponds to the frequency of the fundamental beam). The output photons are spatially separated by a split cavity whose free spectral range is  $2\Delta\omega$ . The idler beam passing through the split cavity is sent to two frequency filtering cavities, and subsequently split into three equal-intensity beams with beamsplitters. The state of each beam is then displaced by a specific amplitude by interfering it with a displacement beam at a mirror of 99% reflectivity. The phase of the displacement is controlled by piezoelectric transducers, and the amplitude of the displacement is controlled by rotating half-wave plates followed by polarization beamsplitters. The idler photons are detected by APDs. When APDs detect photons, they output electronic pulses which are combined into an AND circuit to get threefold coincidence clicks. The *signal* beam is measured by homodyne detection with a local oscillator beam of 10 mW. The homodyne current is sent to an oscilloscope and stored every time coincident clicks happen. The density matrix and Wigner function of the output state are then numerically reconstructed from a set of measured quadratures and phases of the local oscillator beam.

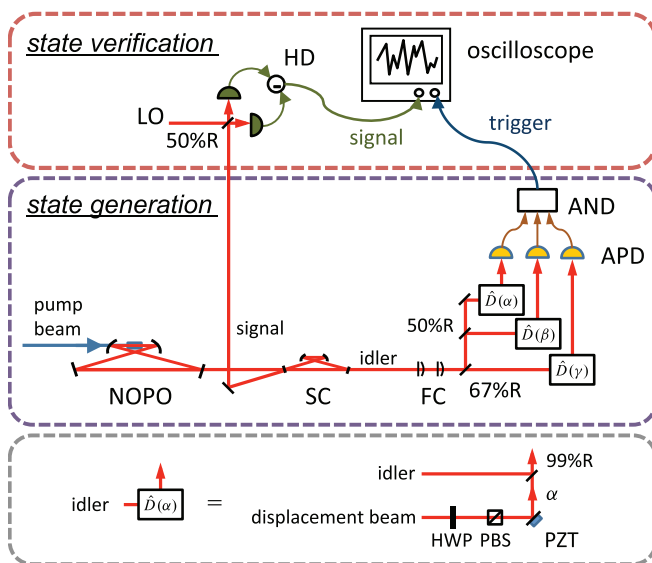


FIG. 1. (Color online) Experimental setup. NOPO, nondegenerate optical parametric oscillator; SC, split cavity; FC, filter cavity; HD, homodyne detector; APD, avalanche photodiode; HWP, half-wave plate; PBS, polarization beamsplitter; PZT, piezoelectric transducer.

### IV. ANALYSIS OF THE EXPERIMENTAL STATE

The reconstructed quantum state, both its density matrix  $\hat{\rho}_{\text{exp}}$  and its Wigner function, is shown in in Fig. 2(a). The

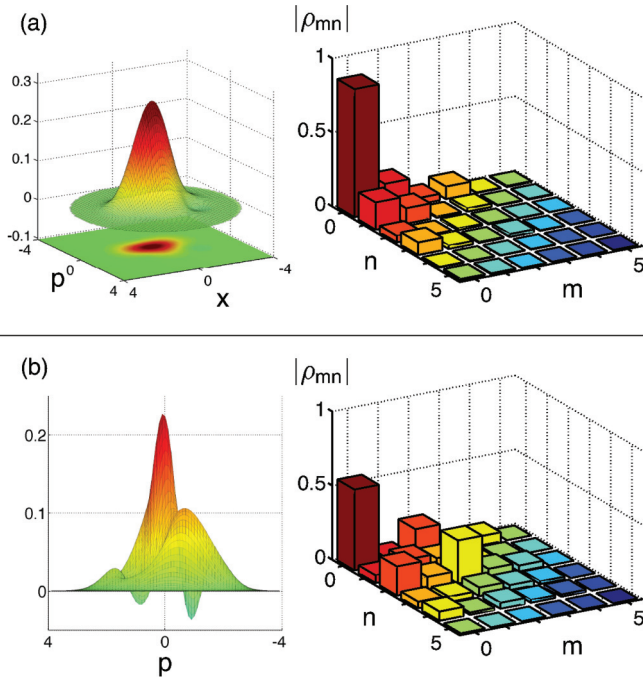


FIG. 2. (Color online) (a) Wigner function and density matrix of the experimentally generated state. (b) Wigner function and density matrix of the experimentally generated state after a single photon is numerically subtracted from the data.

traditional approach to quantifying the quality of a prepared state is by using the fidelity,  $F = \langle \psi_{id} | \hat{\rho} | \psi_{id} \rangle$ . In our case, the generated state has a maximal fidelity of  $F_M = 0.90$  with the ideal state (1) with  $\chi = 0.166$ . However, due to the weak nonlinearity, the same state has fidelity  $F_0 = 0.95$  with the vacuum state. This does not suggest that vacuum is a better cubic state but rather that the fidelity is not a good figure of merit in our case. In order to verify the state, we will therefore need to analyze it more thoroughly and devise new methods.

We can start by confirming the presence of nontrivial superpositions of photon numbers present in (1). The overwhelming influence of vacuum can be removed by applying a virtual single-photon subtraction  $\hat{\rho}_{exp} \rightarrow \hat{\rho}_{1sub} = \hat{a}_{exp} \hat{a}^\dagger / \text{Tr}[\hat{a}_{exp} \hat{a}^\dagger]$ . For the ideal resource state,  $(1 - i\chi \hat{x}^3)|0\rangle$ , this should result in a superposition  $|0\rangle + \sqrt{2}|2\rangle$ , which is a state fairly similar to an even superposition of coherent states and, as such, it should possess several regions of negativity. Thus we can convert the cubic state into a state with well-known properties, which can be easily tested. Figure 2(b) shows the Wigner function and the density matrix of the numerically photon-subtracted experimental state. Notice that two distinctive regions of negativity are indeed present. Moreover, apart from considerations involving specific states, the areas of negativity sufficiently indicate nonclassical behavior of the initial state, as they would not appear if the state was only a mixture of coherent states, which does not produce entanglement when divided on a beamsplitter [28]. The probability of two photons  $p'_2 = 0.29$  is clearly dominating over  $p'_1 = 0.12$  and  $p'_3 = 0.03$ , where  $p'_i = \langle i | \hat{\rho}_{1sub} | i \rangle$ . To show now that Fock states  $|0\rangle$  and  $|2\rangle$  appear in the superposition and not in the mixture, we use the normalized off-diagonal element for states basis  $|\phi\rangle$  and  $|\xi\rangle$ ,  $\mathcal{R}_{\xi,\phi}(\hat{\rho}) = \frac{|\langle \xi | \hat{\rho} | \phi \rangle|^2}{\langle \xi | \hat{\rho} | \xi \rangle \langle \phi | \hat{\rho} | \phi \rangle}$ , which

characterizes the quality of any unbalanced superposition. Since the subtraction preserves the superposition of Fock states,  $\mathcal{R}_{0,2}(\hat{\rho}_{1sub}) = 0.24$  after the subtraction proves the presence of coherent superposition originating from the state  $|1&3\rangle$ . In a similar way we can confirm that the three-photon element is significantly dominant over the two- and four-photon elements. Two virtual photon subtractions transform the state  $\hat{\rho}_{exp} \rightarrow \hat{\rho}_{2sub} = \hat{a}^2 \hat{\rho}_{exp} \hat{a}^{\dagger 2} / \text{Tr}[\hat{a}^2 \hat{\rho}_{exp} \hat{a}^{\dagger 2}]$ , where the single-photon state is present with a probability of  $p''_1 = \langle 1 | \hat{\rho}_{2sub} | 1 \rangle = 0.68$ . In a generated single-photon state this would be a sufficient confirmation that the state cannot be emulated by a mixture of Gaussian states. In our case it is the argument for the strong presence of the three-photon element.

Our analysis confirms presence of the highly nonclassical superposition state  $|1&3\rangle$ , but we also need to demonstrate that the state appears in a superposition with the vacuum state, not just as a part of mixture. For this we look at the normalized off-diagonal element  $\mathcal{R}_{0,1&3}(\hat{\rho}_{exp})$  between the  $|0\rangle$  and  $|1&3\rangle$  for the original (not photon-subtracted) experimental state, which would attain a value of one for the ideal pure state and a value of zero for a complete mixture. In our case the value is  $\mathcal{R}_{0,1&3}(\hat{\rho}_{exp}) = 0.50$ , so the superposition is present, even if it is not perfectly visible due to the effects of noise. More importantly, the element is significantly larger than  $\mathcal{R}_{0,1&3^\perp}(\hat{\rho}_{exp}) = 0.11$ , where  $|1&3^\perp\rangle = (\sqrt{2}|1\rangle - \sqrt{3}|3\rangle)/\sqrt{5}$  is orthogonal to  $|1&3\rangle$ . This shows that the desired and theoretically expected superpositions are dominant.

## V. DETECTING CUBIC NONLINEARITY

We have shown that the state contains the required superpositions, which is a strong argument about the true nature of the state. However, there is also some measure of noise present. It is a valid question, then, whether the state does indeed behave as the cubic state despite the imperfections. The cubic state should be able to drive, even at this elementary level, the cubic gate. One way the cubic gate manifests is observable even at a semiclassical level. For a given quantum state, the cubic nonlinearity transforms the first quadrature moments  $\hat{x}_{in}$  and  $\hat{p}_{in}$  according to  $\langle \hat{x}_{out} \rangle = \langle \hat{x}_{in} \rangle$ ,  $\langle \hat{p}_{out} \rangle = \langle \hat{p}_{in} \rangle + 3\chi \langle \hat{x}_{in}^2 \rangle$ . The first moment of  $\hat{x}$  should be preserved, while the first moment of  $\hat{p}$  should become linearly dependent on the second moment  $\langle \hat{x}^2 \rangle = \text{var}(x) + \langle \hat{x} \rangle^2$ . Note that  $\text{var}(x)$  is a variance of  $\hat{x}$ . If we choose a set of input states with identical variances, there should be observable quadratic dependence of the first moment of  $\hat{p}$  on the first moment of  $\hat{x}$ .

The easiest way the cubic gate can be implemented relies on mixing the prepared ancilla with the target state on a balanced beamsplitter, which is followed by projecting the ancilla onto the quadrature eigenstate  $|x=0\rangle$  by homodyne detection. This is the probabilistic version of the cubic gate [18] and it is similar to using single photons to obtain a probabilistic map [29]. As the set of target states, we will consider coherent states  $|\alpha\rangle$ , where  $0 \leq \alpha \leq 1$ , with first moments  $\langle \hat{x}_{in} \rangle = \sqrt{2}\alpha$  and  $\langle \hat{p}_{in} \rangle = 0$ . The operation, imprinting nonlinearity from the ancillary mixed state  $\hat{\rho}_A$  to the target state  $\hat{\rho}_{in} = |\alpha\rangle\langle\alpha|$ , can be realized by the map

$$\hat{\rho}_{out} = \text{Tr}_A[\hat{U}_{BS} \hat{\rho}_{in} \otimes \hat{\rho}_A \hat{U}_{BS}^\dagger |x=0\rangle_A \langle x=0|], \quad (3)$$

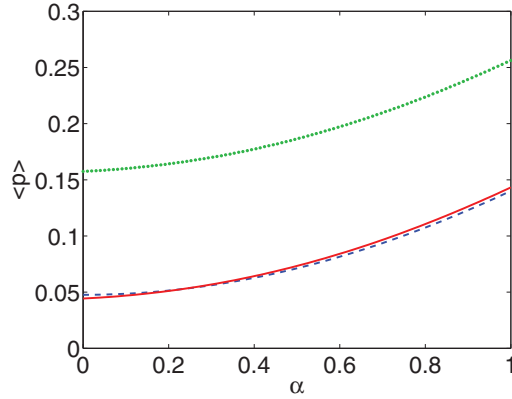


FIG. 3. (Color online) First moment of  $p$  for various coherent states: ideal state with  $\chi = 0.090$  (dashed blue line), experimentally generated state (dotted green line), and experimentally generated state after the suitable displacement  $\Delta p = -0.16$  (solid red line).

where  $\hat{U}_{BS}$  is a unitary operator realizing transformation by a balanced beamsplitter. This map fuses two states with wave functions  $\psi_S(x_S)$  and  $\psi_A(x_A)$  into a state with wave function  $\psi_S(x_S/\sqrt{2})\psi_A(x_S/\sqrt{2})$ . The factor  $\sqrt{2}$  only introduces linear scaling of the measured data and has no influence on any nonlinear properties. Since the imprinting operation uses only Gaussian tools, any non-Gaussian nonlinearity of the transformed state needs to originate in nonlinear properties of the ancillary state  $\hat{\rho}_A$ . We have numerically simulated the procedure, and the behavior of the first moment of quadrature  $\hat{p}$  is plotted in Fig. 3. We can see that the dependence is distinctively quadratic. This behavior is actually in a very good match with that of the ideal cubic state (1) with  $\chi = 0.090$ . They only differ by a constant displacement, which has probably arisen due to experimental imperfections and which can be easily compensated. This showcases our ability to prepare a quantum state capable of imposing high-order nonlinearity in a different quantum state.

We can also attempt to observe the cubic nonlinearity directly, using the density matrix in coordinate representation. In this picture, the continuous density matrix elements are defined as  $\rho(x, x') = \langle x | \hat{\rho} | x' \rangle$ . The cubic nonlinearity is best visible in the imaginary part of the main antidiagonal: for the ideal state  $(1 - i\chi\hat{x}^3)|0\rangle\langle 0|(1 + i\chi\hat{x}^3)$ , the density matrix elements are  $\text{Im}[\rho(x, -x)] = 2\chi x^3 e^{-x^2}$  and the cubic nonlinearity is nicely visible. One problem in this picture is that the cubic nonlinearity can be concealed by other operations. The second-order nonlinearity does not manifest in the imaginary part (not even order nonlinearities do), but a simple displacement can conceal the desired behavior. On the other hand, displacement can be quite straightforwardly

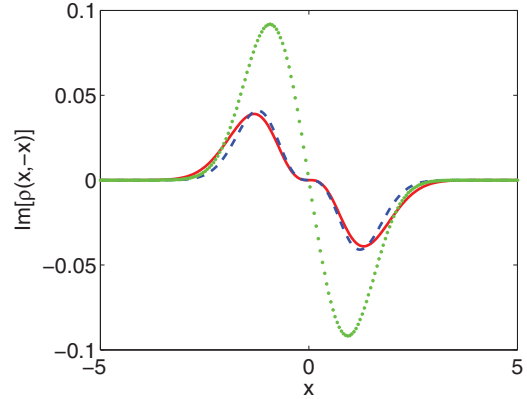


FIG. 4. (Color online) Imaginary parts of the antidiagonal values of coordinate density matrices for the ideal state with  $\chi = 0.090$  (dashed blue line), the experimentally generated state (dotted green line), and the experimentally generated state after the suitable displacement  $\Delta p = -0.17$  (solid red line).

compensated by performing a virtual operation on the data. The comparison of the ideal state, the generated state, and the displaced generated state can be seen in Fig. 4. We can see that although the cubic nonlinearity is not immediately apparent, the suitable displacement can effectively reveal it. This nicely witnesses our ability to conditionally prepare a quantum state equivalent to the outcome of the required higher-order nonlinearity.

## VI. SUMMARY AND OUTLOOK

In conclusion, we have generated a heralded nonclassical non-Gaussian quantum state of light, which exhibits key features of a state produced by unitary dynamics driven by cubic quantum nonlinearity. Our experimental test has demonstrated the feasibility of conditional optical preparation of the ancillary resource state for the cubic measurement-induced nonlinearity. Our analysis has contributed to general understanding of quantum states produced by the higher-order quantum nonlinearities. This understanding is a crucial step towards physically implementing these nonlinearities as a part of quantum information processing, and we expect information regarding the first attempts in this direction to appear soon.

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