

**Direct observation of phase-sensitive Hong-Ou-Mandel interference**

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The quality of individual photons and their ability to interfere are traditionally tested by measuring the Hong-Ou-Mandel photon bunching effect. However, this phase-insensitive measurement only tests the particle aspect of the quantum interference, leaving out the phase-sensitive aspects relevant for continuous-variable processing. To overcome these limitations we formulate a witness capable of recognizing both the indistinguishability of the single photons and their quality with regard to their continuous-variable utilization. We exploit the conditional nonclassical squeezing and show that it can reveal both the particle and the wave aspects of the quantum interference in a single set of direct measurements. We experimentally test the witness by applying it to a pair of independent single photons retrieved on demand.

DOI: [10.1103/PhysRevA.96.033830](https://doi.org/10.1103/PhysRevA.96.033830)**I. INTRODUCTION**

Quantum interference of individual photons is one of the fundamental principles of quantum technology. The ability to recognize whether photons from different sources can interfere is therefore an important one. The basic tests use photon correlation measurements to ascertain the presence of individual photons and two-photon bunching to verify their interference. The experimental test of the bunching effect traditionally employs the Hong-Ou-Mandel interference [1], in which the two individual photons are mixed on a balanced beam splitter and the resulting two-mode field is measured by a pair of photon counters. The bunching manifests as a lack of coincidence counts: If the coincidence probability is below one-half, the interference effect is nonclassical, i.e., incompatible with the description using a mixture of classical waves or classical particles. Recent experiments for a wide range of experimental platforms safely observed low values of coincidence counts and thus demonstrated this fundamental nonclassical aspect [2].

For discrete-variable quantum technology these features are sufficient. However, from the point of view of continuous variables (CVs) and the more general framework of hybrid quantum technology [3–6], these experiments quantify only the particle nature of the interference and ignore the vacuum-related phase-sensitive aspects [7] of the photon bunched state. They also avoid the issues of coupling single photons with CV states, such as mode overlap with the local oscillator. The full picture of two-photon interference can be obtained by replacing the photon counters with homodyne detection and performing a full tomography [8,9]. The reconstructed density matrix can then be analyzed in order to visualize and evaluate the interference caused by the indistinguishability. Drawbacks to this tomographic approach are that it requires a high number of individual measurements and it cannot be used as a direct witness.

Tomographic reconstruction is a process combining information from many incompatible measurements in order to obtain quantities that are not measurable directly, such as

off-diagonal elements of the density matrix. The elements of the reconstructed matrix contain, in principle, full information about the state. However, the information is practically limited due to the unavailability of a complete set of measurements and the need to form prior assumptions about the suitable Hilbert space of the system [10]. For this reason some elements, in particular those related to higher Fock numbers, are difficult to estimate with sufficient precision. This may lead to erroneous or indeterminate conclusions, such as attributing nonclassical properties to the interference of classical signals. Furthermore, the full information provided by the costly tomographic process is often not needed. In this case, use of a specialized and less demanding direct witness with straightforward statistical error analysis is much more effective [11].

Two-photon interference leads to a state that, by suitably conditioning homodyne measurement, can be transformed into a state with quadrature squeezing [7]. This squeezing, although not strong enough to be of much use on its own, can be created neither from classical light nor from noninterfering photons and is therefore an excellent witness of nonclassical and coherence properties. It can be also directly measured. Interestingly, the measurement itself relies only on classical correlations within the state, without exploiting its entanglement [12] or discord [13]. The general witness therefore requires one further condition: The phase of the initial states needs to be randomized. This step, while not affecting single-photon states, cancels the possible false positives. The full phase-sensitive Hong-Ou-Mandel effect therefore serves as a directly observable witness simultaneously for both particle and wavelike aspects of the two-photon interference. It also offers a much more challenging benchmark against which new single-photon sources can be tested.

In this paper we introduce a witness for remotely prepared single-photon states, which summarily assesses their single-photon nature, indistinguishability, and compatibility with hybrid processing circuits. The witness relies on observation of nonclassical quadrature correlations and thus requires only measuring a single two-quadrature joint probability

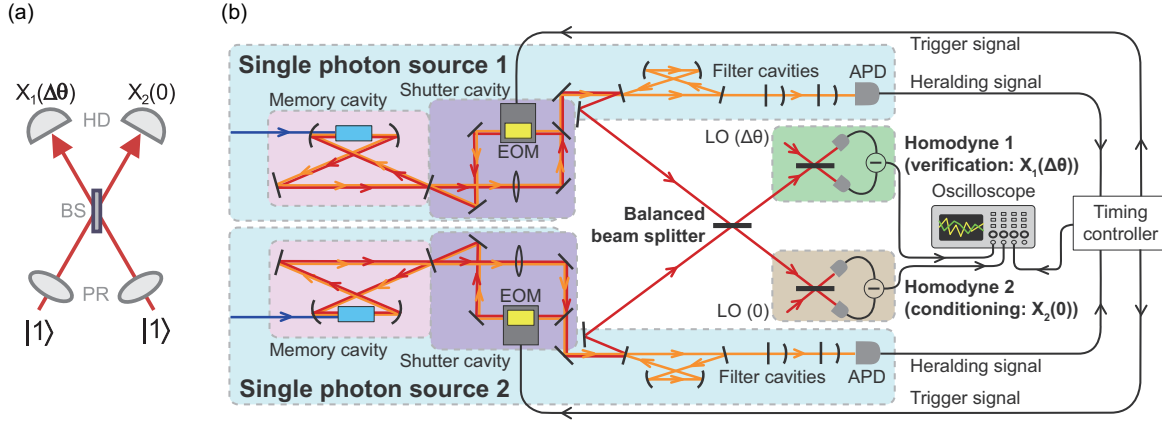


FIG. 1. (a) Conceptual scheme of the witness. The phase randomized (PR) states are mixed on a balanced beam splitter (BS) and two fixed quadratures are then measured by a pair of homodyne detectors (HD). (b) Experimental setup. Two single-photon states are independently generated and stored in the memory cavities. After synchronization, they are released at the same time, allowed to interfere on the balanced beam splitter, and recorded by the pair of homodyne detectors. Here EOM denotes electro-optic modulator, LO local oscillator, and APD avalanche photodiode.

distribution, which makes it significantly less demanding than quantum tomography. For demonstration we apply the witness to a pair of photons individually prepared in timing synchronization cavities [8].

## II. SINGLE-PHOTON WITNESS

In the ideal version of the Hong-Ou-Mandel experiment, the two single photons indistinguishable in all but a single degree of freedom are in the initial state  $|1, 1\rangle$  and get mixed on a balanced beam splitter. This produces an entangled state

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|20\rangle - e^{i\phi}|02\rangle), \quad (1)$$

with  $\phi$  being the relative phase between the two output modes introduced by the beam splitter. The two resulting modes are then subjected to intensity measurements by avalanche photodiodes whose action is described by their positive-operator-valued measure elements  $\Pi_0 = |0\rangle\langle 0|$  and  $\Pi_1 = 1 - \Pi_0$ , which provide us with probabilities  $P_{kj} = \langle \psi | \Pi_k \otimes \Pi_j | \psi \rangle$ ,  $k, j = 0, 1$ . For indistinguishable photons there is a distinct lack of coincidences represented by  $P_{11} = 0$ . On the other hand, for completely distinguishable photons the probability of coincident detection is firmly set to  $P_{11} = 1/2$ . The full Hong-Ou-Mandel measurement is realized by deliberately delaying one of the photons, thus increasing the distinguishability, and measuring the coincidence rates for these different settings. The effect is then quantified in terms of visibility, which is defined as

$$V = \frac{\max P_{11} - \min P_{11}}{\max P_{11} + \min P_{11}}, \quad (2)$$

where the maximization and minimization are performed over a set of different delays. For perfectly indistinguishable (distinguishable) photons the visibility is  $V = 1$  ( $V = 0$ ). The visibility can be also reduced due to imperfection of the single-photon states, caused by the presence of vacuum or higher Fock terms. However, since the vacuum terms do not contribute towards coincidence rates, only the higher Fock terms lead to

a reduction in visibility. This means that the Hong-Ou-Mandel effect is a good indicator of distinguishability of single photons, but it provides limited insight into their quality. Another shortcoming of the Hong-Ou-Mandel effect is related to the technological aspect of the employed single-photon detectors. These particle detectors have a very broad detection spectrum incompatible with the much narrower spectrum of continuous homodyne detectors, which play a crucial role in CV quantum information processing with light [3]. This discrepancy can be removed by using optical filtering, but this comes at the cost of detection efficiency and alteration of the photon statistics. Another issue is the fundamental identity of the single-photon detectors as intensity detectors. As such, the measurement is insensitive to the vacuum and cannot reveal the wave portion of the interference, the relative phase between the modes, related to the phase-sensitive correlations between the field quadratures. As a consequence, with the particle detectors the state (1) cannot be differentiated from a completely mixed state  $(|20\rangle\langle 20| + |02\rangle\langle 02|)/2$ . Needless to say, in CV and hybrid quantum information processing this distinction is quite crucial.

We are therefore looking for a strong unifying witness capable of verifying the quality of single-photon states as well as their indistinguishability and suitability for hybrid processing [4,5]. We also want to observe the interference effect directly, without the need to consider different measurement bases and settings, and subsequent reconstruction of the state's density matrix.

All of these requirements are reconciled in a three-step procedure illustrated in Fig. 1(a), in which the two single-photon states are first phase randomized, then they interfere on a balanced beam splitter, and finally they are subjected to the joint measurement of phase-correlated quadrature operators  $X_1(\theta_1)$  and  $X_2(\theta_2)$ , where  $X_j(\theta) = X_j \cos \theta + P_j \sin \theta$ , with  $j = 1, 2$ . Here  $X_j$  and  $P_j$  obey the canonical commutation relation  $[X_j, P_j] = i$ . Because of the phase insensitivity of initial single-photon states, a common component of phases has no significance. Hence, quadrature operators can be expressed only with the differential component  $\Delta\theta = \theta_1 - \theta_2$

as  $X_1(\Delta\theta)$  and  $X_2(0)$ . Let us look at the ideal scenario with the state (1) and consider a homodyne measurement of mode 2 yielding a value  $x_2$ . This measurement projects the state of the remaining mode 1 into a state proportional to  $e^{i\phi} \langle x_2|2\rangle|0\rangle - \langle x_2|0\rangle|2\rangle$ . This state has zero mean values of the quadrature operators  $X_1$  and  $P_1$ . The remarkable thing is that by choosing some particular values of  $x_2$ , the state in mode 1 is projected into a squeezed state, whose second moment at a certain measurement angle shows a value less than  $1/2$  [7,14]. The witness, confirming that state  $\rho$  was obtained by the interference of two phase-insensitive single-photon states on a balanced beam splitter, can be therefore formulated as

$$E[X_1^2(\Delta\theta)|X_2(0)=x_2] = \frac{\text{Tr}[X_1^2(\Delta\theta) \otimes |x_2\rangle\langle x_2|\rho]}{\text{Tr}[1 \otimes |x_2\rangle\langle x_2|\rho]} < \frac{1}{2}, \quad (3)$$

where  $E[\cdot|C]$  represents the conditional mean value of an operator when condition  $C$  is satisfied. The phase  $\Delta\theta_{\text{sq}}$  that gives the minimum second moment is tied to the phase of the superposition  $\phi$  as  $\Delta\theta_{\text{sq}} = -\phi/2$ . It is important that  $\phi$  is arbitrarily controllable by classical phase locking, so  $\Delta\theta_{\text{sq}}$  is known *a priori* and can be set to an arbitrary value. The procedure therefore requires measuring the statistics of only a single pair of quadrature operators, which makes it much more practical and economical than the full tomography. Witness (3) is never satisfied when the states are distinguishable, classical, or incompatible with the CV part of the experiment. However, the presence of other Fock components can diminish the squeezing and change the range of conditioning values  $x_2$  that reveal it. On the other hand, when the squeezing threshold is reduced below the vacuum level, the witness becomes stricter until eventually it can be satisfied only by pure perfectly interfering single-photon states. This is discussed in greater detail in the Appendix.

The conditioning on  $x_2$  is related to the generalized theory of squeezing extraction presented in Ref. [7], but there are some interesting distinctions. Due to the ideally vanishing nature of the first moments, the condition can be specified with help of the second *noncentral* moment, which makes it unsatisfiable by a lone single photon [15]. This is in contrast to the observing squeezing of the conditional central moment, the variance, which can be achieved even for a single photon interacting with the vacuum and may therefore lead to false positives. However, the conditional squeezing can be considered as a witness for the quality of *single* photons [7]. For example, an imperfect single-photon state represented by the density operator  $\rho = \eta|1\rangle\langle 1| + (1-\eta)|0\rangle\langle 0|$  can be confirmed as nonclassical for an arbitrary positive value of  $\eta$ . The method demonstrates the phase-sensitive continuous-variable nature of the correlations in photon anticorrelation experiments. It is also interesting to point out that when the squeezing is replaced by another nonclassicality witness employing only a single quadrature measurement, such as the one proposed in Ref. [16], no conditional nonclassicality can be detected.

### III. EXPERIMENTAL TEST

The experimental setup for the test of the phase-sensitive Hong-Ou-Mandel effect for two single-photon states is based on our previous experiment reported in Ref. [8] and it is

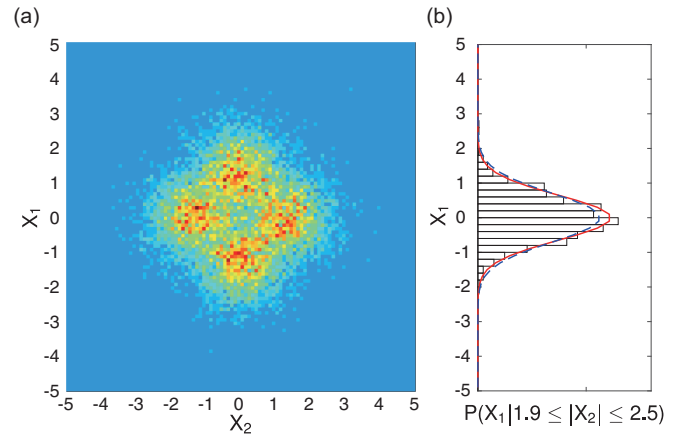


FIG. 2. Experimentally measured quadrature distribution of the state after interference with  $\Delta\theta = \Delta\theta_{\text{sq}}$ . (a) Joint probability distribution  $P(X_1, X_2)$ . (b) Normalized quadrature distribution  $P(X_1)$ . An occurrence rate of  $X_1$  with the window  $1.9 \leq |X_2| \leq 2.5$  is shown in histogram. Lines are Gaussian distributions with the variance of conditioned  $P(X_1)$  (red solid line) and that of vacuum fluctuation for reference (blue dashed line), respectively.

shown in Fig. 1(b). Each of the two photon sources is based on the heralded scheme with parametric down-conversion contained in an optical cavity. Photon pairs are generated from the continuous-wave pump light at 430 nm at random times. One photon of the pair immediately leaves the cavity and is detected by an avalanche photodiode, thus heralding the presence of another 860-nm photon stored in the cavity. A technique involving concatenated shutter cavity [17,18] can be then used to release the second photon at a desired time. This allows timing synchronization of the two generated photons and enables their interference on the balanced beam splitter. With this technique, the simultaneous emission events occur 20 times in a second with 400-ns synchronization window and we gathered quadrature data of 13 037 events. Since the single-photon states are generated independently, their phase is random and needs no further randomization. The state after interference is measured by phase-sensitive homodyne detection in both output modes of the balanced beam splitter: One mode is used for conditioning to generate squeezing and the other mode is used for the verification of squeezing in the conditioned state. The relative phase between a local oscillator and the output mode at both homodyne detections, and the relative phase of two local oscillators that is equivalent to  $\Delta\theta$  are actively stabilized. The quantum efficiency of homodyne detectors and the visibility between a signal beam and the local oscillator are both more than 99%. Any correction of losses is not applied to the results below.

Figure 2(a) shows the joint measured distribution of quadratures  $X_1$  and  $X_2$  of the two modes emerging from the beam splitter with  $\Delta\theta = \Delta\theta_{\text{sq}}$ . The quadratures were locked against each other, but their relation to the outside reference varied over time. This effectively resulted in a statistics that is equivalent to measurement without an outside phase reference. We will therefore omit the phase dependence from now on. We can see that the quadrature data are distinctively correlated in a manner consistent with the target state. In accordance

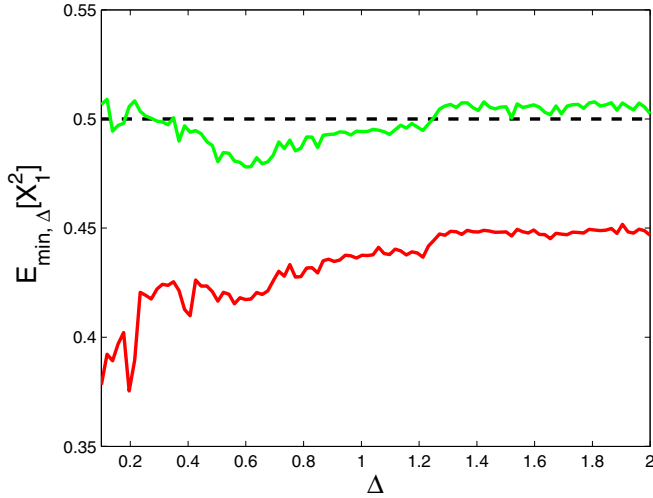


FIG. 3. Conditioned second moment  $E_{\min, \Delta}[X_1^2]$  (red line) and its upper confidence band (green line) relative to the width of the postselection interval  $\Delta$ . The position of the conditioning interval given by the measured value of  $X_2$  was optimized over in order to obtain minimal certifiable variance given by the upper confidence interval.

with the proposed witness, we are looking for a quantitative indicator of the Hong-Ou-Mandel coherence in the form of the second conditional moment  $E[X_1^2|X_2 = x_2]$ . The theory asks for conditioning on a single measurement result. Since this is impossible, we have relaxed the requirement for the condition and accepted situations in which the value of quadrature  $X_2$  fell into a bin of width  $\Delta$ ,  $x_2 - \Delta/2 \leq X_2 \leq x_2 + \Delta/2$ . Due to the symmetrical nature of the distribution, we also used both positive and negative values in the postselection. An example of the conditional quadrature distribution of  $X_1$  is shown in Fig. 2(b). The conditioning window, denoted by the set  $B$ , was set to  $1.9 \leq |X_2| \leq 2.5$  and it picked 1011 of the initial 13 037 data points. The conditional variance was found to be  $E[X_1^2|X_2 \in B] = 0.42$ , which is smaller than the vacuum fluctuation variance of 0.5.

Let us now discuss the results in more detail, giving more attention to possible statistical errors, and make sure that the observed squeezing is significant. For that we need to construct the confidence band for the estimated variance. Under the assumption that the conditional distributions are approximately Gaussian, we used the Monte Carlo method to simulate 1000 runs of the experiment, estimated their moments, and evaluated their statistics. The standard deviation of the estimated variances depends on the number of data points and therefore on the position and width of the postselection interval. A narrow interval generally yields the lowest possible moment values, but they come at the cost of a low number of data points and, consequently, less certainty about the validity of the result. The best certifiable violation is obtained when the upper boundary of the confidence band reaches its minimum value. Figure 3 shows the second moment with its upper confidence band estimated from the measured data. For all considered values of  $\Delta$  we have optimized over the values  $x_2$  marking the center of the interval to obtain the minimum of the upper confidence band boundary. The plotted moment can

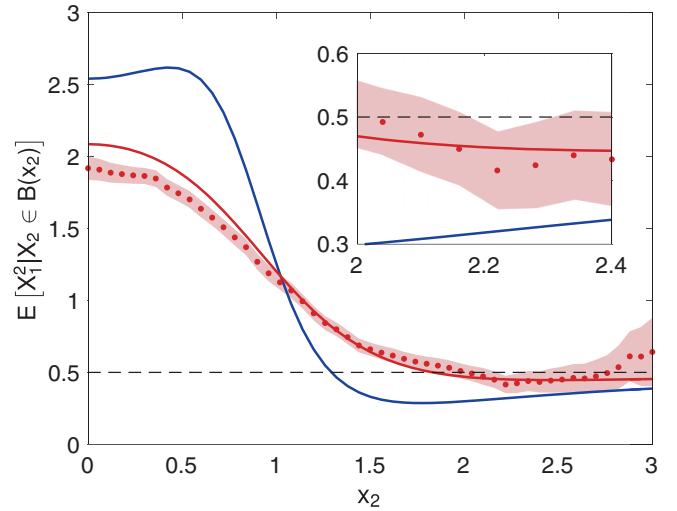


FIG. 4. Conditional second moment  $E[X_1^2|X_2 \in B(x_2)]$ , where  $B(x_2)$  is the set of values satisfying  $x_2 - \Delta/2 \leq |X_2| \leq x_2 + \Delta/2$ . The width of the postselection interval was chosen to be  $\Delta = 0.6$ . The blue dashed line marks the theoretical prediction for the interference of two ideal single-photon states. The red solid line shows the prediction for interference of two imperfect photons produced by the cavities. Red dots represent experimental data with the light red zone marking the  $3\sigma$  confidence band. The inset shows detailed behavior in the area with the highest certifiable squeezing.

therefore be expressed as

$$E_{\min, \Delta}[X_1^2] = \min_{x_2} \left\{ E \left[ X_1^2 \left| x_2 - \frac{\Delta}{2} < |X_2| < x_2 + \frac{\Delta}{2} \right. \right] \right\}. \quad (4)$$

We have obtained the lowest certifiable variance for  $\Delta = 0.6$ .

The conditional variances for this scenario are shown in Fig. 4. We can see that the measured data closely follow the curve predicted from knowledge of the two independent single-photon states, which was obtained by their individual tomographic reconstructions [8]. This shows that the amount of squeezing is limited mainly by the undesired vacuum component of the single photons and not their coherence. Furthermore, the squeezing is observable even if we take the three-standard-deviation-wide confidence band into account. This confirms that the squeezing is not a static artifact and demonstrates both the nonclassical nature of the two initial single-photon states and their mutual coherence. This first application of the proposed witness also shows its sensitivity to small imperfection of these components. Note that the visibility of the traditional Hong-Ou-Mandel interference estimated from the tomography data would be  $V = 0.7$  in our experiment.

#### IV. SUMMARY

We have suggested a phase-sensitive Hong-Ou-Mandel effect as a witness for recognizing the presence and non-classicality of individual photons, their ability to quantum mechanically interfere, and their compatibility with hybrid



quantum processing [3]. These aspects are detected jointly; a lack of any one of them will lead to diminishing performance and separating them is not straightforward. However, since all of these properties are needed for hybrid processing, the ability to test them at the same time is highly practical. The suggested witness requires joint measurement of only a single pair of commuting quadrature operators. There is no need for tomography and sets of incompatible measurements, which reduces the demands on the length of the measurement by more than an order of magnitude. The employed measurement does not rely on entanglement [12] or discord [13]. We have applied the witness to the pair of on-demand generated photons and demonstrated both their nonclassical nature and their mutual coherence. The witness can be directly applied to other experimental platforms where generation of interfering Fock states plays an important role, such as atomic ensembles [19], optomechanics [20,21], and superconducting circuits [22,23].

#### ACKNOWLEDGMENTS

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#### APPENDIX

The proposed witness takes advantage of nonclassical properties of the interfering quantum states. As such, higher Fock components of the initial states, which are nonclassical on their own, can affect the outcome. Since we consider phase randomized states with a density matrix diagonal in the Fock representation,  $\rho_{1,2} = \sum_k P_k |k\rangle\langle k|$ , we can analyze the effect by looking at the witness value for initial states  $|m,n\rangle$ , because the final witness value will be obtained as their linear combination. The conditional second moments for states  $|m,n\rangle$ , where  $m,n = 0,1,2$ , are displayed in Fig. 5. We can see that states  $|1,0\rangle$  and  $|2,0\rangle$  never lead to squeezing and numerics confirmed this behavior for states up to  $|10,0\rangle$ . This demonstrates that even when the two states are imperfect, their coherence is an absolutely essential condition for observation of any squeezing.

We can also see that apart from the state  $|1,1\rangle$ , squeezing can be also obtained for states  $|1,2\rangle$  and  $|2,2\rangle$ . However, the minimum conditioning value for which the squeezing is observable is larger than that of the state  $|1,1\rangle$ . Combinations of higher Fock terms have not been plotted, but the trend remains the same: For some states squeezing can be observed, but it is always for progressively larger values of  $x_2$ . This can be used to derive an even stronger form of the witness, which eliminates contributions of these higher terms. For example, squeezing found for conditioning value  $1.23 < x_2 < 1.75$  can only be a consequence of the state  $|1,1\rangle$ .

For an explicit demonstration let us consider a subset of states with at most two photons. Such states have a density matrix  $\rho_1 = \rho_2 = p_0|0\rangle\langle 0| + p_1|1\rangle\langle 1| + (1-p_0-p_1)|2\rangle\langle 2|$ .

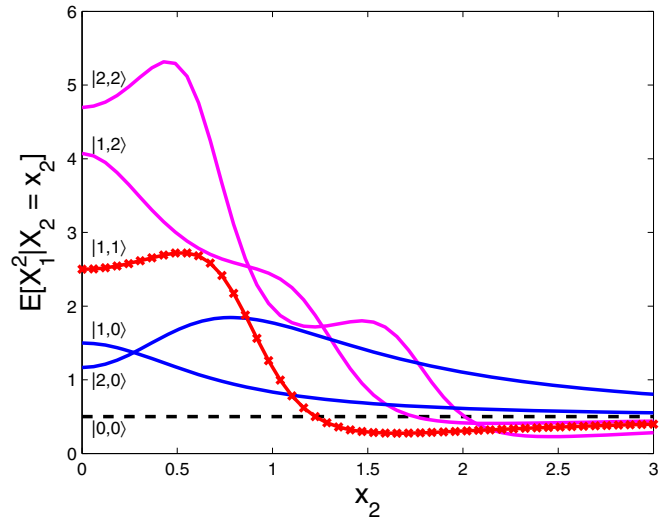


FIG. 5. Conditional second moment for different input states.

Figure 6 shows the range of parameters in which squeezing can be detected for two different conditioning values. Here  $x_2 = 1.5$  corresponds to the strong ideal witness, which detects only single-photon contributions, while  $x_2 = 2.2$  is the value that was used in the experimental implementation. We can see that the first case restrictively singles out only states with a high single-photon component, while the second one allows also for a two-photon contribution. However, the second one is also more forgiving when it comes to losses causing a higher proportion of the vacuum, which is the feature we can take advantage of if we admit the realistic experimental assumption that  $p_2 \ll 1$ .

The amount of observed squeezing can be used for quantitative statements about the quality of the single photons and their distinguishability. Lowering the required moment value

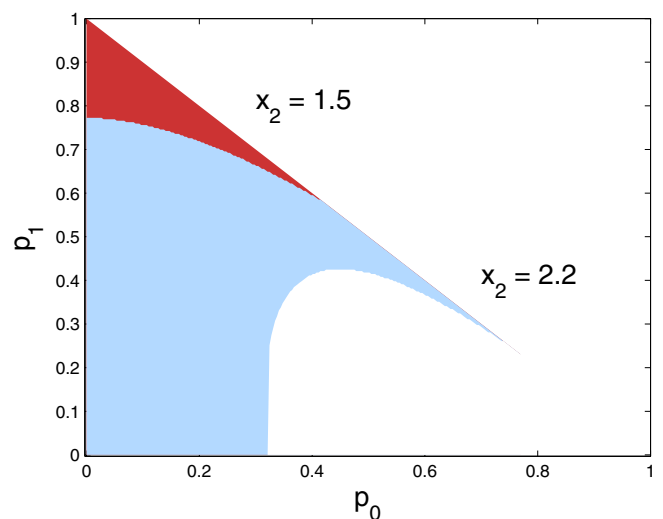


FIG. 6. Range of parameters for states  $\rho_1 = \rho_2 = p_0|0\rangle\langle 0| + p_1|1\rangle\langle 1| + (1-p_0-p_1)|2\rangle\langle 2|$ , in which squeezing can be witnessed. For the conditioning value  $x_2 = 2.2$  squeezing can be observed in both the light blue and dark red areas. For the conditioning value  $x_2 = 1.5$  squeezing can be observed only in the dark red area.

reduces the range of parameters for which the behavior can be witnessed. A sufficiently low value then detects only pure perfectly interfering single-photon states. Alternatively, larger observed squeezing allows for wider confidence intervals and thus for more valid statements.

In summary, observation of squeezing always guarantees that the two states are nonclassical and coherent. Observation of squeezing for particular values of  $x_2$  can provide further information and even verify the presence of specific Fock components.

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- [15] The single photon divided on a beam splitter and conditioned by homodyne detection can be always expressed as  $c_0|0\rangle + c_1|1\rangle$ . In this state,  $\langle X^2 \rangle = 0.5 + |c_1|^2$  and the moment is never squeezed.
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