Control Theory Applied to Accelerated Gradient Optimization Algorithms

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This presentation will describe some recent work in which control theory methods were used to analyze some gradient based optimization methods. Gradient based optimization methods are widely used in the area of quantum control. The heavy ball method is analyzed using the discrete time circle criterion for a class of non-convex cost functions and it shown that convergence could only be guaranteed for cost functions with a condition number which less than a fixed maximum value. Then the coefficients of the original heavy ball algorithm were modified so that the circle criterion could be used to guarantee convergence for all cost functions in the class being considered, at the expense of a slightly slower convergence rate. Also, an additional term was added to the heavy ball algorithm to obtain an improved guaranteed convergence rate. In addition, a control theory result on optimal gain margin was used to prove that the original heavy ball method has the fastest convergence rate of any linear gradient base optimization method for strictly convex quadratic cost functions. In addition, the case of quadratic cost functions with moving optimal points was addressed by introducing integral action into the optimization algorithms.