Comparison of estimation limits for quantum two-parameter estimation

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Measurement estimation bounds for local quantum multiparameter estimation, which provide lower bounds on possible measurement uncertainties, have so far been formulated in two ways: by extending the classical Cramér–Rao bound (e.g., the quantum Cramér–Rao bound and the Nagaoka Cramér–Rao bound) and by incorporating the parameter estimation framework with the uncertainty principle, as in the Lu–Wang uncertainty relation. In this work, we present a general framework that allows a direct comparison between these different types of estimation limits. Specifically, we compare the attainability of the Nagaoka Cramér–Rao bound and the Lu–Wang uncertainty relation, using analytical and numerical techniques. We show that these two limits can provide different information about the physically attainable precision. We present an example where both limits provide the same attainable precision and an example where the Lu–Wang uncertainty relation is not attainable even for pure states. We further demonstrate that the unattainability in the latter case arises because the figure of merit underpinning the Lu–Wang uncertainty relation (the difference between the quantum and classical Fisher information matrices) does not necessarily agree with the conventionally used figure of merit (mean-squared error). The results offer insights into the general attainability and applicability of the Lu–Wang uncertainty relation. Furthermore, our proposed framework for comparing bounds of different types may prove useful in other settings.

References

[1] S. K. Yung, L. O. Conlon, J. Zhao, P. K. Lam, and S. M. Assad, *Comparison of estimation limits for quantum two-parameter estimation*, Physical Review Research 6, 033315 (2024).