Scaling of overheads in large-scale fault-tolerant quantum computation

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Fault-tolerant quantum computation (FTQC) is a method of protecting computing steps from errors which occur continually due to faults in the physical devices. Threshold theorems assure that once the quality of physical devices is improved beyond a certain threshold, quantum computation with any number of logical qubits and any number of steps is possible. One of the important questions toward the realization of a large-scale quantum computer is the cost of FTQC, namely, how much increase one should pay in terms of space and time to fight against errors and what is the best structure of FTQC to reduce the costs. The number of physical qubits per logical qubit is called space overhead, and the number of physical steps per logical step is called time overhead. How these two numbers scale with the size of computational task to solve is a good measure of how efficient an FTQC method is for large-scale quantum computation.

The earliest proposal of FTQC achieving a threshold theorem was based on concatenated codes. Tiny error correcting codes to protect a single logical qubit is nested again and again to protect it against many physical errors. In contrast to this early approach, the recent research trend considers a single 'big' code, such as a surface code and more generally a quantum LDPC code, to protect against many errors. The latte approach has a striking feature that one can increase the number of logical qubits as the size of the code, leading to a constant space overhead.

In this talk, I will discuss our recent progress in both approaches. We were able to prove threshold theorems even when we include the scaling of classical computation time used in the execution of FTQC protocols. For concatenated codes, we showed that a constant space overhead is also achievable in this approach by using a series of small codes protecting multiple logical qubits rather than a single one. For LDPC codes, we drastically improved its time overhead scaling to lift the apparent trade-off relation between space and time overheads. We believe that our new findings have shed a new light on the discussion on various FTQC methods in terms of efficiency for large-scale quantum computation.

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